DIMENSIONLESS RELATIONS FOR ZERO-CONSUMPTION ELECTRIC ARCS IN A CHANNEL

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Limiting dimensionless relations are presented for the characteristics of an arc in a channel when the consumption of the plasma-generating gas approaches zero. An extensive comparison of calculation with experiment is carried out.

In [1], generalized expressions were obtained for calculating the characteristics of a radiating electric arc in a plasmatron channel on the basis of the most general anisotropic model. The model is based on a power-law approximation for the dependence of electrical conductivity on the function of thermal conductivity with different exponents for the longitudinal and transverse components of this function, whose product is used in a procedure for separation of variables.

Of greatest interest is the asymptotic case, which is realized most completely in so-called weakly ventilated or zero-consumption electric arcs, when the characteristics of the arc depend only on the radius virtually along the entire length of the channel. For this kind of arc in the atmospheres of various gases, a large volume of experimental evidence has been accumulated; moreover, there are a number of works dealing with the volt–ampere characteristics and temperature fields under identical operating conditions. These experiments were conducted to determine the thermophysical properties of plasma. They form a good base for checking generalized formulas obtained theoretically. On the other hand, the procedure, for generalization of experimental data widely used at present for integrted volt–ampere characteristics turned out to be inedequate in the case of local $E-I$-dependences for various gases. This motivated the present investigation.

When $\bar{z}/\mathrm{Pe}\bar{r}_*^2 \to \infty$ and $\alpha_n^2\bar{z}/\mathrm{Pe} \to \infty$, from the expressions given in [1] we obtain the following formulas for the radius of an electrically conductive column, the distribution of the thermal conductivity function in electrically conductive and nonconductive zones, the electric-field intensity, and the heat flux on the wall in this limiting case:

$$\bar{r}_* = \exp\left[\frac{1}{J_1(\mu_1)\,\mu_1}\,\frac{\Delta S_1}{\Delta S_0}\left(\frac{1}{4\pi^2 k J_1^2(\mu_1)\,\bar{r}_*^2\,(1 + Q_0 R^2 \bar{r}_*^2/\Delta S_0\,\mu_1^2)}\,\frac{I^2}{R^2\sigma_0\Delta S_0}\right)^{-\frac{1}{n+1}}\right], \tag{1}$$

$$\frac{\Delta S_{\mathrm{I}}(\bar{r})}{\Delta S_0} = \left(\frac{1}{4\pi^2 k J_1^2(\mu_1)\,\bar{r}_*^2\,(1 + Q_0 R^2 \bar{r}_*^2/\Delta S_0\,\mu_1^2)}\,\frac{I^2}{R^2\sigma_0\Delta S_0}\right)^{\frac{1}{n+1}} J_0(\mu_1\bar{r}/\bar{r}_*)\,. \tag{2}$$

$$\frac{\Delta S_{\mathrm{II}}(\bar{r})}{\Delta S_0} = -\frac{\Delta S_1}{\Delta S_0}\,\frac{\ln(\bar{r}/\bar{r}_*)}{\ln\bar{r}_*}, \tag{3}$$

$$\frac{ER^2\sigma_0}{I} = \frac{\mu_1}{2\pi k J_1(\mu_1)\,\bar{r}_*^2}\left(\frac{1}{4\pi^2 k J_1^2(\mu_1)\,\bar{r}_*^2\,(1 + Q_0 R^2 \bar{r}_*^2/\Delta S_0\,\mu_1^2)}\,\frac{I^2}{R^2\sigma_0\Delta S_0}\right)^{-\frac{n}{n+1}}, \tag{4}$$

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$$\frac{q_1 R}{\Delta S_0} = J_1(\mu_1)\,\mu_1 \left(\frac{1}{4\pi^2 k J_1^2(\mu_1)\,\bar{r}_*^2\,(1 + Q_0 R^2 \bar{r}_*^2/\Delta S_0 \mu_1^2)}\ \frac{I^2}{R^2 \sigma_0 \Delta S_0} \right)^{\frac{1}{n+1}} . \tag{5}$$

Using the symbols Po, Pe, K_Q, K_S, Π_E, and Nu for the criteria and similarity numbers, we can represent expressions (1)-(5) in a more compact form with numerical coefficients:

$$\bar{r}_* = \exp\left[-\,0.8 K_S \left(\frac{\text{Po}}{10.6 k \bar{r}_*^2 (1 + K_Q \bar{r}_*^2)} \right)^{-\frac{1}{n+1}} \right], \tag{6}$$

$$\overline{\Delta S}_{\text{I}}(\bar{r}) = \left(\frac{\text{Po}}{10.6 k \bar{r}_*^2 (1 + 0.17 K_Q \bar{r}_*^2)} \right)^{\frac{1}{n+1}} J_0(2.4\bar{r}/\bar{r}_*)\,, \tag{7}$$

$$\overline{\Delta S}_{\text{II}}(\bar{r}) = K_S \frac{\ln(\bar{r}/\bar{r}_*)}{\ln \bar{r}_*}\,, \tag{8}$$

$$\Pi_E = \frac{0.74}{k\bar{r}_*^2} \left(\frac{\text{Po}}{10.6 k \bar{r}_*^2 (1 + 0.17 K_Q \bar{r}_*^2)} \right)^{-\frac{n}{n+1}}, \tag{9}$$

$$\text{Nu} = 1.25 \left(\frac{\text{Po}}{10.6 k \bar{r}_*^2 (1 + 0.17 K_Q \bar{r}_*^2)} \right)^{\frac{1}{n+1}} . \tag{10}$$

From Eqs. (6)-(10) we can see that all the characteristics of the arc are the same functions of three criteria: Po, K_Q, and K_S. In dimensional form they are written, respectively, as $I^2/R^2(I/R)$, $R^2(R)$, $-\Delta S_1$.

Calculations cause certain inconveniences, because formula (1) or (6) for the radius of the electrically conductive zone cannot be written in explicit form. However, the inverse function Po $(\bar{r}_*, K_Q, K_S)$, in which the quantities K_Q and K_S play the role of parameters, can be expressed explicitly:

$$\text{Po} = [10.6 k \bar{r}_*^2 (1 + 0.17 K_Q \bar{r}_*^2)] \left(\frac{0.8 K_S}{\ln \frac{1}{\bar{r}_*}} \right)^{n+1} . \tag{11}$$

After substitution of Eq. (11) into Eqs. (7), (9), and (10), we obtain a still more compact and convenient form of the expressions for the characteristics of the arc:

$$\overline{\Delta S}_{\text{I}}(\bar{r}) = \frac{0.8 K_S}{\ln \frac{1}{\bar{r}_*}} J_0(2.4\bar{r}/\bar{r}_*)\,, \tag{12}$$

$$\Pi_E = \frac{0.74}{k\bar{r}_*^2} \left(1.25 \frac{\ln \frac{1}{\bar{r}_*}}{K_S} \right)^{n}, \tag{13}$$

$$\mathrm{Nu} = \frac{\mathrm{K}_S}{\ln \frac{1}{\bar{r}_*}}. \tag{14}$$

Formulas (11)-(14) and formula (8) (whose form remained unchanged) represent a system of dimensionless equations, in which the arguments Po, $\bar{r}$, and the functions $\Delta S_{\mathrm{I}}/\Delta S_{00}$, $\Delta S_{\mathrm{II}}/\Delta S_{00}$, Π_E, Nu are related parametrically through the radius of the electrically conductive zone $\bar{r}_*$ and the prescribed parameters K_Q and K_S.

Another parametric system of dimensionless equations can be obtained if instead of $\bar{r}_*$ we use $\overline{\Delta S}_{00} = \Delta S_{00}/\Delta S_0$ as a parameter known a priori. For this purpose, we express $\bar{r}_*$ in terms of $\overline{\Delta S}_{00}$, assuming in Eq. (12) that $\bar{r} = 0$. This yields

$$\bar{r}_* = \exp\left(-0.8\mathrm{K}_S/\overline{\Delta S}_{00}\right). \tag{15}$$

Substituting Eq. (15) into Eqs. (8), (11)-(14), we obtain

$$\mathrm{Po} = \left\{10.6k \exp\left(-1.6\mathrm{K}_S/\overline{\Delta S}_{00}\right) \left[1 + 0.17\mathrm{K}_Q \exp\left(-1.6\mathrm{K}_S/\overline{\Delta S}_{00}\right)\right]\right\} \overline{\Delta S}_{00}^{\,n+1}, \tag{16}$$

$$\overline{\Delta S}_{\mathrm{I}}(\bar{r}) = \overline{\Delta S}_{00} J_0\left[2.4\,\bar{r}/\exp\left(-0.8\mathrm{K}_S/\overline{\Delta S}_{00}\right)\right], \tag{17}$$

$$\overline{\Delta S}_{\mathrm{II}}(\bar{r}) = -\left(1.25\overline{\Delta S}_{00} \ln \bar{r} + \mathrm{K}_S\right), \tag{18}$$

$$\Pi_E = (0.74/k)\left(\mathrm{K}_S/\overline{\Delta S}_{00}\right)^n \exp\left(\mathrm{K}_S/\overline{\Delta S}_{00}\right), \tag{19}$$

$$\mathrm{Nu} = 1.25\overline{\Delta S}_{00}. \tag{20}$$

In expressions (15)-(20) $\overline{\Delta S}_{00}$ can be replaced by $\overline{\Delta S}_{\mathrm{av}}$ averaged over the cross-section of the electrically conductive column, since for the Bessel temperature profile $\overline{\Delta S}_{00} = 2.32\overline{\Delta S}_{\mathrm{av}}$.

At large values of I/R, if the radius of the electrically conductive zone in a zero approximation is assumed to be equal to unity, then from Eq. (6) we obtain an expression for $\bar{r}_*$ in explicit form

$$\bar{r}_* = \exp\left[-0.8\,\mathrm{K}_S\left(\frac{\mathrm{Po}}{10.6k\,(1 + 0.17\mathrm{K}_Q)}\right)^{-\frac{1}{n+1}}\right]. \tag{21}$$

After substitution of Eq. (21) into Eqs. (7)-(10) and elimination of $\bar{r}_*$, we obtain the following approximate relations for $\bar{r}_*$ close to unity:

$$\overline{\Delta S}_{\mathrm{I}}(\bar{r}) = \left(\frac{\mathrm{Po}}{10.6k}\right)^{\frac{1}{n+1}}\left\{1 + 0.17\mathrm{K}_Q \exp\left[-1.6\mathrm{K}_S\left(\frac{\mathrm{Po}}{10.6k\,(1 + 0.17\mathrm{K}_Q)}\right)^{-\frac{1}{n+1}}\right]\right\}^{-\frac{1}{n+1}} \times$$

$$\times \exp\left[\frac{1.6\mathrm{K}_S}{n+1}\left(\frac{\mathrm{Po}}{10.6k\,(1 + 0.17\mathrm{K}_Q)}\right)^{-\frac{1}{n+1}}\right] J_0\left(2.4\,\bar{r}/\bar{r}_*\right), \tag{22}$$

$$\overline{\Delta S}_{\mathrm{II}}(\bar{r}) = \mathrm{K}_S\left(\frac{\ln \bar{r}}{0.8\mathrm{K}_S\left(\dfrac{\mathrm{Po}}{10.6k\,(1 + 0.17\mathrm{K}_Q)}\right)^{-1/(n+1)}} + 1\right). \tag{23}$$

TABLE 1. Parameters for Approximation of Thermophysical Properties

Parameter	Dimension	Plasma-generating gas			
		air [2-4]	nitrogen [6, 7]	argon [8, 10]	helium [11, 12]
Range of variation of T	10^3 K	4–15	6–15	3–15	4–15
T_*	10^3 K	7	6.83	7	7.47
S_*	10^3 W/m	5.42	5.76	0.71	7.95
T_1	K	600	600	600	600
S_1	W/m	30	26	20	150
ΔS_1	10^3 W/m	−5.4	−5.73	−0.69	−7.8
T_0	10^3 K	10	10	10	10
ΔS_0	10^3 W/m	5.88	6.14	0.99	5.47
σ_0	$10^3\ (\Omega\cdot\mathrm{m})^{-1}$	2.15	3.12	2.5	0.054
n	–	1.25	0.827	0.54	2.75
k	–	0.8	0.99	1.49	0.98
$1/(n+1)$	–	0.44	0.547	0.65	0.27
$n/(n+1)$	–	0.56	0.453	0.35	0.73
b	$10^5\ \mathrm{m}^{-2}$	2.85	12.2	32 or 185 $\overline{\Delta S_{00}}^{1.04}$	0.547

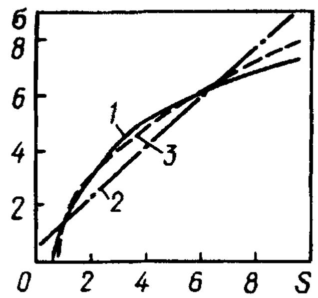


Fig. 1. Approximation of the dependence of electric conductivity on the thermal conductivity function for argon: 1) calculation according to the data of [8]; 2) linear approximation; 3) power-law approximation. $P = 0.1$ MPa; $\sigma\cdot 10^3\ (\Omega\cdot\mathrm{m})^{-1}$; S, kW/m.

$$\Pi_E = \frac{0.74}{k}\left(\frac{\mathrm{Po}}{10.6k}\right)^{-\frac{n}{n+1}}\left\{1 + 0.17\mathrm{K}_Q \exp\left[-1.6\mathrm{K}_S\left(\frac{\mathrm{Po}}{10.6k\,(1+0.17\mathrm{K}_Q)}\right)^{-\frac{1}{n+1}}\right]\right\}^{-\frac{n}{n+1}} \times$$

$$\times \exp\left[\frac{n+2}{n+1}\mathrm{K}_S\left(\frac{\mathrm{Po}}{10.6k\,(1+0.17\mathrm{K}_Q)}\right)^{-\frac{1}{n+1}}\right], \tag{24}$$

$$\mathrm{Nu} = 1.25\left(\frac{\mathrm{Po}}{10.6k}\right)^{\frac{1}{n+1}}\left\{1 + 0.17\mathrm{K}_Q \exp\left[-1.6\mathrm{K}_S\left(\frac{\mathrm{Po}}{10.6\,k\,(1+0.17\mathrm{K}_Q)}\right)^{-\frac{1}{n+1}}\right]\right\}^{-\frac{1}{n+1}} \times$$

$$\times \exp\left[\frac{1.6\mathrm{K}_S}{n+1}\left(\frac{\mathrm{Po}}{10.6k\,(1+0.17\mathrm{K}_Q)}\right)^{-\frac{1}{n+1}}\right]. \tag{25}$$

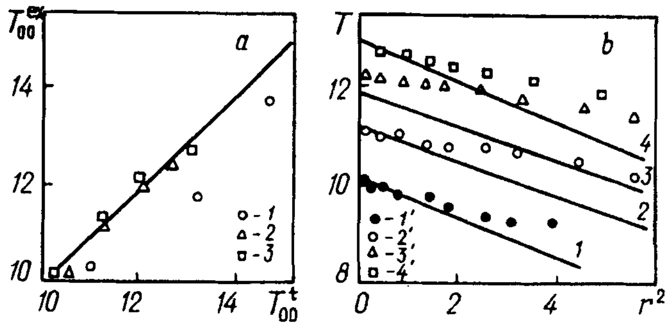


Fig. 2. Comparison of theoretical (lines) and experimental (points) data [9] for the axial temperature (a) and temperature profiles (b) of an argon arc: a) 1) $R = 2$ mm; 2) 3; 3) 4; b) 1, 1′) $I = 40$ A; 2, 2′) 80; 3, 3′) 120; 4, 4′) 200. $R = 4$ mm, $P = 0.1$ MPa, r^2, mm^2, T_{00}^{ex}, T_{00}^{t}, kK.

Since the model involves a power-law approximation of the thermophysical properties of plasma-generating substances, preliminary processing of data by their properties is required. The following approximating formulas were used:

$$\sigma/\sigma_0 = (\Delta S/\Delta S_0)^n , \tag{26}$$

$$Q = b\Delta S . \tag{27}$$

The value of S_* entering into ΔS at the boundary of the electrically conductive zone was determined beforehand by linear approximation of the dependence $\sigma(S)$ using the least squares method. Processing was carried out in the temperature range of from $(3–6)\cdot 10^3$ K to $15\cdot 10^3$ K, which is typical for the overwhelming majority of the arcs investigated. The necessary thermophysical properties are taken from [2-12]. The values of the approximation parameters are given in Table 1. Figure 1 illustrates the accuracy of approximation of the dependence $\sigma(S)$.

Because of the large nonlinearity of the function $Q(\Delta S)$ for argon, the approximation of its linear dependence at $b = \text{const}$ turned out to be insufficient. Therefore, we used approximation (27) at $b = 185\Delta S_{00}^{1.04}$. The value of ΔS_{00} on the axis in the first approximation was calculated from Eq. (2) or (7) and (12) at $b = 3.2\cdot 10^6$ m^{-2}.

The theoretical dependences were compared with experimental data for various gases in both physical and generalized coordinates.

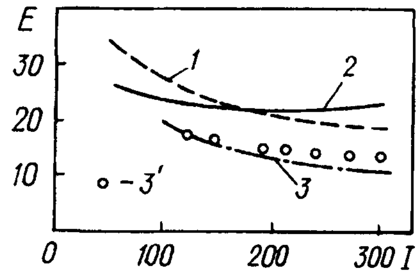
Figure 2 presents a comparison of the theoretical calculation from formulas (1) and (2) or correspondingly (6) and (7), or (11) and (12) and the experimental data of [9] for the profiles of the temperature and its axial value. The maximum calculation error is ~13%.

The nongeneralized $E–I$ characteristics for air and helium are given in Fig. 3, and the generalized $E–I$ characteristic for air, nitrogen, argon, and helium, in Fig. 4. Calculations were performed from expressions (11) and (13), which correspond to Eqs. (1) and (4) or (6) and (9). It is evident that the greatest error (~230%) is noted for an argon arc in a large-diameter channel (R = 10.5 mm).

For the remaining experimental data the deviation from the theoretical curve does not exceed ±29%. The considerable error for large-diameter channels seems to be caused by ignoring radiation reabsorption.

Checking of the generalized dimensionless relations for four gases and comparison of calculation with experiment by two parameters (T and E) show that unification of formulas does not lead to an unacceptable decrease in calculation accuracy.

Generalized function (9) together with (6) describes both the ascending branches of the $E–I$ characteristics, which are typical for argon in the compared limits of the change in the arc parameters [9], and the descending branches, which are most typical for helium [14] (Fig. 3). This is associated with the value of the exponent n in approximation (25).



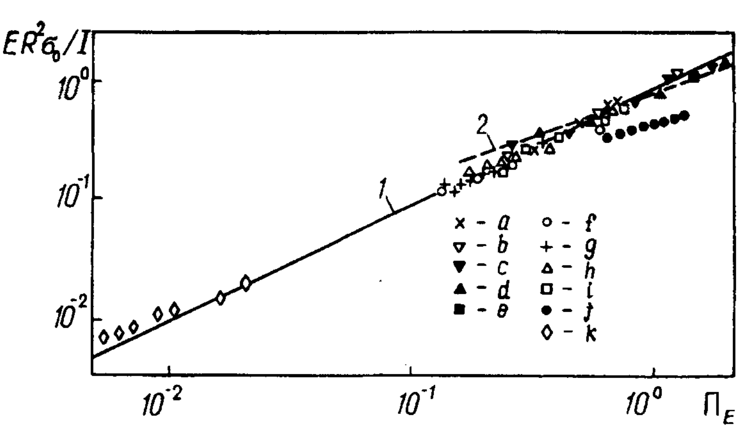


Fig. 3. Ungeneralized $E-I$ characteristics of electric arcs: 1, 3) theory; 2, 3′) experiment [13, 14]; 1, 2) air; $R = 2.5$ mm; 3, 3′) helium, $R = 5$ mm. $P = 0.1$ MPa; E, W/cm; I, A.

Fig. 4. Generalized $E-I$ characteristic: 1) theory; 2) experiment for air [13], $R = 2.5$ mm; points denote experiment for nitrogen [6], argon [9], and helium [14]. Nitrogen: a) $R = 1$ mm; b) 1.5; c) 2; d) 2.5; e) 3. Argon: f) $R = 2$ mm; g) 2.5; h) 3; i) 4; j) 10.5. Helium: k) $R = 5$ mm. $P = 0.1$ MPa.

We will give an approximate condition of transition from descending to ascending $E-I$ characteristic. Assuming for simplicity that $\bar{r}_* = \text{const}$, and having taken in Eq. (4) the derivative dE/dI and equated it to zero, we obtain $n - 1 = 0$. At $n = 1$ within the framework of the simplifications adopted, the $E-I$ characteristic has a minimum. Since the exponent is not constant, but depends on the approximation range of $\sigma(\Delta S)$ and changes for different gases from several units to several tens of unity fractions, then the $E-I$ characteristics pass through a point at which the intensity is independent of the current. In a first approximation the calculated $E-I$ dependence has a descending character for $n > 1$ and an ascending one for $n < 1$. According to Table 1, in the range $T = (4-15) \cdot 10^3$ K the first case is most typical for helium (because of the great difference from unity), while the second case is most typical for argon.

Actually, owing to the fact that, according to Eq. (1), $r_* = f(I/R)$, the minimum of the $E-I$ characteristic is reached at any value of n. Therefore, in spite of the fact that for nitrogen $n < 1$, the $E(I)$ correlation has a descending character within the indicated range of parameters, and an increase in intensity begins at somewhat larger values of I.

One obtains expressions with the same exponents for different gases in the case of identical parameters of the approximation of the corresponding plasma properties. But the accuracy of the generalized formulas is worsened.

By virtue of the above, within the scope of the existing ideas, we may speak only of the approximate similarity of electric arcs. The accuracy of generalized expressions depends on the universality of the parameters for the approximation of thermophysical properties; this universality is determined in turn by the quantity and composition of the group of the gases studied, as well as by the range of the arc parameters.

The technique made it possible for the first time to generalize by the same dependence experimental data for the local volt-ampere characteristics of a zero-consumption arc within a considerable range of parameters and thermophysical properties: $I = 20-300$ A, $R = 2-5$ mm, air, nitrogen, argon, and helium.

NOTATION

C_p, isobaric heat capacity; σ, electrical conductivity; λ, thermal conductivity; T, temperature; P, pressure; E, electric-field intensity; I, current strength; r, z, radial and longitudinal coordinates; $S = \int_0^T \lambda dT$, thermal conductivity function; G, gas flow rate; Q, density of volumetric radiation; q, heat flux density; R, channel radius;

$\bar{r} = r/R$; $\bar{z} = z/R$; $\Delta S = S - S_*$; $\overline{\Delta S} = \Delta S/\Delta S_0$; $b = Q_0/\Delta S_0$; J_0, J_1, Bessel functions; μ_1, α_n, roots of characteristic equations; k, n, constants; $\mathrm{Pe} = GC_{p0}/\pi R\lambda_0$, Peclet number; $\mathrm{Po} = I^2/R^2\sigma_0\Delta S_0$, Pomerantsev number; $K_Q = Q_0R^2/\Delta S_0$, radiation criterion; $K_S = -\Delta S_1/\Delta S_0$, parametric criterion; $\mathrm{Nu} = q_1R/\Delta S_0$, modified Nusselt number; $\Pi_E = ER^2\sigma_0/I$, similarity number for determining E. Subscripts and superscripts: *, boundary of the electrically conductive zone; 0, base value; 1, value on the wall; 00, axial value; I, II, electrically conductive and nonconductive zone, respectively; av, value averaged over the cross-section of the electrically conductive zone; t, theory; ex, experiment.

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